

Extra Practice Problems 6

Here's one final set of practice problems for the second midterm exam. We've released solutions along with this handout, but please *don't read them* until you've actually attempted these problems!

Eulerian and Hamiltonian Graphs

Recall that a graph is called *Eulerian* if there is a cycle that passes through every *edge* exactly once and *Hamiltonian* if there is a cycle that passes through every *node* exactly once.

- i. Find a graph that is Hamiltonian but not Eulerian.
- ii. Find a graph that is Eulerian but not Hamiltonian.

More Fun With Friends and Strangers

Suppose you have a 17-clique (that is, an undirected graph with 17 nodes where there's an edge between each pair of nodes) where each edge is colored one of *three* different colors (say, red, green, and blue). Prove that regardless of how the 17-clique is colored, it must contain a blue triangle, a red triangle, or a green triangle. (*Hint: Use the theorem on friends and strangers.*)

Bijections and Induction

Let $f: \mathbb{N} \rightarrow \mathbb{N}$ be a function. We'll say that f is *linearly bounded* if $f(n) \leq n$ for all $n \in \mathbb{N}$.

Prove that if $f: \mathbb{N} \rightarrow \mathbb{N}$ is linearly bounded and is a bijection, then $f(n) = n$ for all $n \in \mathbb{N}$. (*Hint: You might find induction useful here.*)

DFAs, NFAs, and Regular Expressions

Let $\Sigma = \{a, b\}$ and consider the following language over Σ :

$$L = \{ w \in \Sigma^* \mid \text{some letter in } w \text{ appears at least four times} \}$$

- i. Design an NFA for L .
- ii. Write a regular expression for L .